

# Condensate of low dimensional charged Bose disks in a uniform magnetic field

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The Bose condensation of a stack of low dimensional disks which is composed of a noninteracting charged Bose gas in a uniform magnetic field is studied. A statistical approach with density of states at noninteger dimensions are applied for the system. The condensate fraction of the disk system in a uniform magnetic field is calculated. The stack of low dimensional charged Bose disks is found not to share the condensate behavior of the traditional BCS superfluids.

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It is known that two fermions can be coupled to form a pair which behaves like a spinless boson. Many bosonic pairs form a kind of charged Bose gas, and it has also been known that the condensation of the Bose system could be a reliable candidate for superfluidity.<sup>1,2</sup> It has been reported that a dominant contribution to the superfluid density of liquid helium-4 in films and porous media originates from the geometric structure, such as the non-integer dimensionality of the samples.<sup>3</sup> The dimensionality represents the effects in the measure of disorderness in terms of the connectivity of the system.<sup>4</sup>

We are interested in the superfluid density of a stack of noninteracting low dimensional charged Bose disks (CBD) in a uniform magnetic field. We assume that the distance between any two fermions of a pair is large enough to neglect the Coulomb interaction. Condensate density plays a key role in the superfluid density. The low dimensional disk is defined as an object which has a dimension of between 1 and 2. It could be a very thin, spotted system with negligible thickness to maintain the dimensionality less than 2.

However, a system which has a dimension of equal or less than 2 can not produce any nonzero superfluid density. It also has been known that an ideal charged Bose gas in two dimensions (2D) can not be condensed even under a magnetic field because of the one dimensional (1D) character of particle motion within the lowest Landau level.<sup>5,6</sup> On the other hand, if we pile up the low dimensional disks in parallel, this gives an extra dimension to the perpendicular axis.<sup>7</sup> The whole dimension of the stack then becomes between 2 and 3, which is large enough to create a nonzero superfluid density. We apply a simple statistical approach for the  $D$  noninteger dimensions.

The theory we use begins from the noninteracting Bose gas in disk dimensions. It is uniform in disk directions. This is then extended to a charged system such as the bipolaronic method for the condensate density.<sup>1,2</sup> The partition function of the system is given as

$$\ln \mathcal{Q}(z, T, D) = - \int_0^\infty d\epsilon \rho_D(\epsilon) \ln(1 - ze^{-\epsilon/T}) - \ln(1 - z), \quad (1)$$

where  $z$  is the fugacity defined by  $z = e^{-|\mu|/T}$ , and  $\epsilon_{\mathbf{p}} = p^2/2m$  is taken for the neutral system. The term  $\rho_D(\epsilon)$  is the density of states at  $D$ -dimension, the spectral dimension,<sup>4</sup> and plays a key role in our analysis. The spectral dimension of the stack of disks depends very much on how the disks are interconnected.

For a neutral and uniform system it is given as<sup>3,4</sup>

$$\rho_D(\epsilon) = a_D \epsilon^{D/2-1}, \quad (2)$$

where  $a_D$  is a  $D$ -dimensional coefficient given by  $a_D = \Gamma(\frac{D}{2})^{-1} \left(\frac{m^*}{2\pi}\right)^{D/2}$ . Here,  $\Gamma$  is the Gamma function and  $m^*$  is the effective mass of a pair. We set  $\hbar = c = k_B = 1$  for convenience, and unit volume is assumed.

The average number of particles is obtained from Eq. (1)

$$n = z \frac{\partial \ln \mathcal{Q}}{\partial z} = \int_0^\infty d\epsilon \frac{\rho_D(\epsilon)}{z^{-1}(e^{\epsilon/T} - 1)} + n_0, \quad (3)$$

where  $n_0 = n_{\mathbf{p}=0}$ . Next, substituting the  $D$ -dimensional density of states from Eq. (2) into Eq. (3), the condensate fraction is obtained as<sup>3,4,8</sup>

$$\begin{aligned} \frac{n_0}{n} &= 1 - v \int_0^\infty d\epsilon \frac{\rho_D(\epsilon)}{z^{-1}(e^{\epsilon/T} - 1)} \\ &= 1 - \left\{ \int_0^\infty d\epsilon \frac{\epsilon^{D/2-1}}{e^{\epsilon/T} - 1} \right\}_{T_c}^{-1} \int_0^\infty d\epsilon \frac{\epsilon^{D/2-1}}{e^{\epsilon/T} - 1} \\ &= 1 - \left\{ \frac{T}{T_c(D)} \right\}^{D/2}, \end{aligned} \quad (4)$$

where

$$T_c(D) = \frac{2\pi}{m^* v^{2/3}} \frac{1}{\zeta(\frac{D}{2})^{2/D}}. \quad (5)$$

Here,  $v$  is the volume density and  $\zeta$  is the Riemann-Zeta function. The  $z = 1$  limit is taken for the condensation. Note that  $\int_0^\infty dx x^{D/2-1}/(e^x - 1) = \Gamma(D/2)\zeta(D/2)$ .

The critical temperature in Eq. (5),  $T_c$ , corresponds to the BEC transition temperature. It is rewritten as a function of  $T_c^b$  for the bulk ( $D = 3$ ) as

$$T_c(D) = \frac{1.897}{\zeta(\frac{D}{2})^{2/D}} T_c^b, \quad (6)$$

where  $\zeta(\frac{3}{2})^{2/3} = 1.897$ . It can be readily shown that Eq. (6) satisfies both the ideal thin limit ( $D = 2$ ) and the bulk limit ( $D = 3$ ). Note that the transition would not occur for the 2D limit since  $T_c \sim |\frac{D}{2} - 1|$  as  $D$  approaches to 2.<sup>9</sup>

The CBD model in a uniform magnetic field is now extended using this new  $D$ -dimensional density of states,  $\rho_D(\epsilon, H)$ . Our  $D$ -dimensional system,  $2 < D < 3$ , is composed of  $(D - 1)$ -dimensional planes and an additional dimension which is parallel to the magnetic field. A uniform magnetic field is applied perpendicular to the direction of the disks. The new density of states in  $D$ -dimensions is derived from the Landau quantization law.<sup>2,10</sup> The  $(D - 1)$ -dimensional degeneracy is

$$\rho_{D-1}(\epsilon)\omega_H = \frac{1}{\Gamma(\frac{D-1}{2})} \left(\frac{m^*}{2\pi}\right)^{\frac{D-1}{2}} \epsilon^{\frac{D-1}{2}-1} \omega_H, \quad (7)$$

where  $\omega_H = 2eH/m^*$  is the cyclotron frequency. Therefore,  $\rho_D(\epsilon, H)$  is obtained as

$$\begin{aligned} \rho_D(\epsilon, H) &= \rho_{D-1}(\epsilon)\omega_H \sum_{n,p_z} \delta(\epsilon - \epsilon_{n,p_z}) \\ &= \frac{1}{\sqrt{\pi}\Gamma(\frac{D-1}{2})} \left(\frac{m^*}{2\pi}\right)^{\frac{D}{2}} \epsilon^{\frac{D-1}{2}} \omega_H \\ &\times \sum_{n=0}^{\infty} \frac{1}{\sqrt{\epsilon - (n + \frac{1}{2})\omega_H}}. \end{aligned} \quad (8)$$

The condensate fraction for the charged system is then obtained as

$$\begin{aligned} \frac{n_0}{n} &= 1 - \left\{ \int_0^\infty d\epsilon \frac{\rho_D(\epsilon, H)}{e^{\epsilon/T} - 1} \right\}_{T_c}^{-1} \int_0^\infty d\epsilon \frac{\rho_D(\epsilon, H)}{e^{\epsilon/T} - 1} \\ &= 1 - \left\{ \frac{T}{T_c(D)} \right\}^{D/2} \frac{A(T, \omega_H)}{A(T_c, \omega_H)}. \end{aligned} \quad (9)$$

$A(T, \omega_H)$  is defined by

$$A(T, \omega_H) = \int_{x_0}^\infty dx \frac{x^{\frac{D-1}{2}}}{e^x - 1} \sum_{n=0}^{\infty} \frac{1}{\sqrt{x - (n + \frac{1}{2})\frac{\omega_H}{T}}}, \quad (10)$$

where  $x_0 = (n + \frac{1}{2})\frac{\omega_H}{T}$ . Here, we are considering the range:  $\omega_H/T \ll 1$ .  $A(T, \omega_H)$  itself can diverge, but  $n_0/n$  in Eq. (9) does not.

The condensate density of the charged Boson model contains one additional factor of  $A$  which gives the effect

of the field over that of the neutral model in Eq. (4). It is plotted in FIG. 1, at various disks, as a function of normalized temperature  $T/T_c^b$ . The temperature  $T_c$  is converted with the help of Eq. (6).

We find that the field strongly effects the condensate fraction of the noninteracting CBD. Also, we see the higher the disk dimension, the larger the condensate fraction. The most noticeable effect is that the condensate fraction increases as the magnetic field is increased. This means that the CBD of the low dimensional disks does not share the condensate behavior with the conventional BCS superfluids of antiferromagnetism. This is not surprising because the system we have discussed is a noninteracting charged system, whereas real superconducting films are interacting charged systems. Furthermore, the dimension of superconducting disk is not low dimensional but quasi-two dimensional which is greater than 2. However, if we consider a ferromagnetic material or superconductor which is composed of many disks, we suggest the possibility that it could be modeled by a composition of low dimensional disks of noninteracting bosonic pairs.

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FIG. 1. The condensate fraction of the charged Bose disks in a uniform magnetic field. The solid line is for field free bosons, the dashed line is for  $\omega_H/T_c^b = 10^{-3}$ , and the dotted line is for  $\omega_H/T_c^b = 10^{-2}$ . (a) When  $D=3.0$  (the dimension of a disk is 2.0). (b) When  $D=2.6$  (the dimension of a disk is 1.6).



